New Periodic Three-Body Orbits

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Here we present the initial conditions for new periodic three-body orbits with zero angular momentum. The masses of the three bodies are chosen to be equal and the initial configuration is set to be (see Figure 1):

$$\begin{aligned} &x_1(0) = -1, \;\; x_2(0) = 0, \;\; x_3(0) = 1, \\ &y_1(0) = y_2(0) = y_3(0) = 0, \\ &\dot{x}_1(0) = \dot{x}_3(0) = p_1, \;\; \dot{x}_2(0) = -2p_1, \\ &\dot{y}_1(0) = \dot{y}_3(0) = p_2, \;\; \dot{y}_2(0) = -2p_2. \end{aligned}$$

In this way, the space of initial conditions is restricted to a two-dimensional subspace, and can be parameterized by the initial velocities $p_1 = \dot{x}_1(0)$ and $p_2 = \dot{y}_1(0)$.

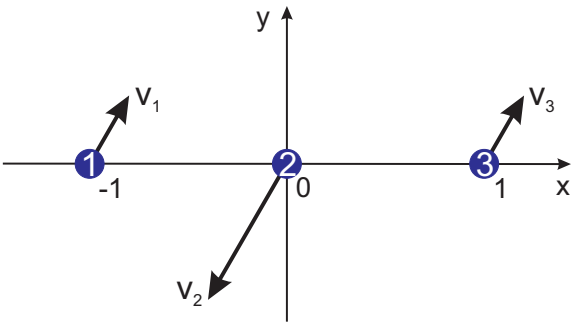

Figure 1: Initial configuration. $\mathbf{v}_1 = \mathbf{v}_3 = -\frac{1}{2}\mathbf{v}_2$

The shapes of these periodic orbits and more details can be found on our website [1]. The orbits are divided into several sequences according to their topologies. All initial conditions (p_1, p_2) and periods T are given in units where the gravitational constant G and the mass of each body m are set to 1. Results for different values of G and m can be obtained by scaling. Further solutions can be obtained by varying the angular momentum L and mass ratios m_1/m_2 and m_1/m_3.

Table I: Initial conditions for periodic three-body orbits in Sequence I (butterfly I).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
I.2.A	0.306893	0.125507	6.234671	[3]
I.2.B	0.392955	0.097579	7.003707	[3]
I.5.A	0.411293	0.260755	20.749072	[6]
I.8.A	0.412103	0.283384	34.247046	[6]
I.9.A	0.402712	0.210016	34.711515	[6]
I.11.A	0.415251	0.291346	47.925856	[6]
I.12.A	0.408211	0.243685	48.486856	[6]
I.13.A	0.399129	0.184708	48.667390	[6]
I.14.A	0.415169	0.295341	61.323559	[6]
I.16.A	0.404132	0.219164	62.446820	[6]
I.17.A	0.397220	0.169198	62.627594	[6]
I.18.A	0.413537	0.271006	76.028536	[6]
I.19.A	0.407376	0.238843	76.221970	[6]
I.20.A	0.401559	0.202266	76.401096	[6]
I.21.A	0.396058	0.158601	76.592744	[6]
I.22.A	0.409622	0.251696	89.986698	[6]
I.23.A	0.404679	0.222598	90.181106	[6]
I.24.A	0.399807	0.189807	90.356363	[6]
I.25.A	0.395290	0.150852	90.562075	[6]

Table II: Initial conditions for periodic three-body orbits in Sequence II (dragonfly).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
II.4.A	0.080584	0.588836	21.272338	[3]
II.6.A	0.186238	0.578714	33.641422	[6]
II.8.A	0.144812	0.542898	38.062055	[6]
II.11.A	0.184279	0.587188	63.534359	[3]
II.14.A	0.108253	0.609812	82.135651	[6]
II.14.B	0.074732	0.567936	68.991156	[6]
II.15.A	0.049051	0.590194	79.152719	[6]
II.15.B	0.058685	0.560793	71.733095	[6]
II.15.C	0.047547	0.564659	72.400492	[6]
II.15.D	0.179107	0.572603	81.599501	[6]
II.16.A	0.073903	0.619865	95.810473	[6]
II.16.B	0.155182	0.579290	86.875231	[6]
II.17.A	0.061053	0.609177	96.873248	[6]
II.17.B	0.050367	0.570341	83.684255	[6]
II.17.C	0.179557	0.581300	95.474382	[6]
II.17.D	0.170632	0.580400	94.199557	[6]

Table III: Initial conditions for periodic three-body orbits in Sequence III (yin-yang).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
III.3.A.α	0.513938	0.304736	17.328810	[3]
III.3.A.β	0.282699	0.327209	10.963252	[3]
III.9.A.α	0.513150	0.289437	50.407145	[6]
III.9.A.β	0.276237	0.331714	32.840840	[6]
III.12.A.α	0.416822	0.330333	55.789329	[3]
III.12.A.β	0.417343	0.313100	54.208001	[3]
III.13.A.α	0.416444	0.336397	63.406504	[6]
III.13.A.β	0.415819	0.306804	60.150751	[6]
III.15.A.α	0.414396	0.339223	70.492561	[6]
III.15.A.β	0.417701	0.303455	66.751098	[6]
III.15.B.α	0.513063	0.296863	85.129064	[6]
III.15.B.β	0.282036	0.325643	54.639089	[6]
III.15.C.α	0.516228	0.311409	88.440089	[6]
III.15.C.β	0.280396	0.329229	54.820022	[6]
III.16.A.α	0.427659	0.340300	80.146699	[6]
III.16.A.β	0.429098	0.299359	74.845173	[6]
III.18.A.α	0.413720	0.341698	84.855117	[6]
III.18.A.β	0.414643	0.301216	79.283684	[6]
III.19.A.β	0.418091	0.299900	86.360447	[6]
III.21.A.β	0.418259	0.299482	92.981286	[6]

Table IV: Initial conditions for periodic three-body orbits in Sequence IVa (moth I).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
IVa.2.A	0.464445	0.396060	14.894306	[3]
IVa.4.A	0.439166	0.452968	28.669273	[3]
IVa.4.B	0.382604	0.459000	25.050583	[6]
IVa.6.A	0.429090	0.475313	42.829966	[6]
IVa.8.A	0.536917	0.453083	77.510884	[6]
IVa.8.B	0.556677	0.434231	79.212485	[6]
IVa.8.C	0.559188	0.427252	78.086940	[6]
IVa.8.D	0.443173	0.492707	62.038492	[6]
IVa.8.E	0.445695	0.492500	62.522933	[6]
IVa.8.F	0.401574	0.490047	54.087338	[6]
IVa.8.G	0.396528	0.492290	53.746595	[6]
IVa.8.H	0.405043	0.492281	55.038964	[6]
IVa.8.I	0.536369	0.370817	60.487172	[6]
IVa.8.J	0.470929	0.463385	61.415988	[6]
IVa.8.K	0.581298	0.389502	81.710548	[6]
IVa.9.A	0.405276	0.496191	62.299938	[6]
IVa.9.B	0.409717	0.439222	54.057257	[6]
IVa.10.A	0.401289	0.498542	68.553666	[6]
IVa.10.B	0.504456	0.397379	70.834775	[6]
IVa.11.A.α	0.395852	0.503779	75.243301	[6]
IVa.11.A.β	0.397055	0.499666	74.404171	[6]
IVa.11.B	0.397055	0.499666	74.404149	[6]
IVa.12.A	0.382405	0.500500	77.873097	[6]
IVa.12.B	0.396136	0.507599	82.909589	[6]
IVa.12.C	0.398039	0.503891	82.249270	[6]
IVa.12.D	0.564500	0.370600	98.867451	[6]
IVa.12.E	0.535402	0.348429	83.984194	[6]
IVa.13.A	0.399096	0.508000	90.359659	[6]
IVa.13.B	0.391290	0.508391	88.515262	[6]
IVa.13.C	0.403099	0.502600	89.642862	[6]
IVa.14.A	0.387473	0.508716	94.120858	[6]
IVa.14.B	0.396148	0.508693	96.420396	[6]
IVa.14.C	0.448047	0.459645	96.839126	[6]
IVa.14.D	0.454267	0.455164	97.362979	[6]
IVa.15.A	0.391105	0.500608	98.676046	[6]
IVa.15.B	0.428275	0.464017	98.649459	[6]
IVa.16.A	0.220100	0.486000	75.547159	[6]

Table V: Initial conditions for periodic three-body orbits in Sequence IVb (butterfly III).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
IVb.3.A	0.405916	0.230163	13.867124	[3]
IVb.6.A	0.414913	0.274619	27.664651	[6]
IVb.7.A	0.398044	0.176138	27.823417	[6]
IVb.9.A	0.414349	0.288103	41.126151	[6]
IVb.11.A	0.395637	0.154450	41.788445	[6]
IVb.14.A	0.401619	0.204794	55.529126	[6]
IVb.14.B	0.403492	0.204300	55.701690	[6]
IVb.15.A	0.415696	0.296400	68.068094	[6]
IVb.15.B	0.413501	0.296800	67.787046	[6]
IVb.15.C.α	0.349985	0.250800	56.840777	[6]
IVb.15.C.β	0.454337	0.228394	68.037708	[6]
IVb.15.D	0.395205	0.142197	55.819348	[6]
IVb.15.E	0.393926	0.142656	55.708470	[6]
IVb.15.F	0.394561	0.142429	55.762418	[6]
IVb.15.G	0.314095	0.244821	60.422077	[6]
IVb.15.H	0.473552	0.210104	78.867863	[6]
IVb.17.A	0.475795	0.160305	79.997226	[6]
IVb.18.A	0.399573	0.188035	69.512255	[6]
IVb.18.B	0.321226	0.204691	62.087458	[6]
IVb.19.A	0.393934	0.134728	69.738915	[6]
IVb.20.A	0.408594	0.245877	83.105868	[6]
IVb.21.A	0.403244	0.213486	83.291527	[6]
IVb.22.A	0.299506	0.292406	80.368724	[6]
IVb.22.B	0.302916	0.197067	73.206425	[6]
IVb.23.A	0.410511	0.256588	96.866250	[6]
IVb.23.B	0.450288	0.099987	92.182051	[6]
IVb.23.C	0.393556	0.129337	83.719536	[6]
IVb.23.D	0.308989	0.157106	74.825736	[6]
IVb.24.A	0.350112	0.079339	79.474771	[3]
IVb.25.A	0.401353	0.200855	97.245642	[6]
IVb.26.A	0.396987	0.167167	97.432196	[6]
IVb.27.A	0.393295	0.125347	97.701289	[6]

Table VI: Initial conditions for periodic three-body orbits in Sequence IVc (moth III).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
IVc.5.A	0.383444	0.377364	25.839237	[3]
IVc.8.A	0.519680	0.353304	57.993976	[6]
IVc.8.B	0.519632	0.353330	57.988255	[6]
IVc.8.C	0.302616	0.357685	37.177247	[6]
IVc.12.A.α	0.463804	0.357385	68.646244	[6]
IVc.12.A.β	0.429325	0.373739	64.873313	[6]
IVc.12.B	0.417326	0.293300	54.853118	[6]
IVc.13.A	0.410016	0.253400	55.377026	[6]
IVc.15.A	0.417304	0.296100	68.275322	[6]
IVc.15.B	0.338374	0.276396	61.185117	[6]
IVc.15.C	0.339476	0.276300	61.283862	[6]
IVc.16.A	0.471671	0.252351	77.839533	[6]
IVc.16.B	0.411718	0.266600	69.037431	[6]
IVc.16.C	0.412798	0.266401	69.174994	[6]
IVc.17.A	0.413909	0.344900	82.863509	[6]
IVc.17.B	0.476139	0.268743	85.381116	[6]
IVc.18.A	0.415114	0.297900	81.302669	[6]
IVc.19.A	0.403994	0.366295	93.785491	[6]
IVc.19.B.α	0.406956	0.354377	94.774607	[6]
IVc.19.B.β	0.390504	0.351287	90.899441	[6]
IVc.21.A	0.415603	0.298500	94.653253	[6]
IVc.22.A	0.406702	0.282083	95.077858	[6]

Table VII: Initial conditions for periodic three-body orbits in Sequence V (figure 8).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
V.1.A	0.347113	0.532727	6.325897	[2]
V.1.B	0.339393	0.536191	6.290748	[2]
V.4.A.α	0.557809	0.451774	39.595045	[6]
V.4.A.β	0.181943	0.514806	18.142107	[6]
V.5.A	0.568991	0.449429	51.964502	[5]
V.7.A	0.203492	0.518113	32.849071	[4]
V.7.B	0.456811	0.540331	64.833294	[4]
V.7.C	0.255431	0.516386	35.043086	[4]
V.7.D	0.410355	0.551985	57.545253	[4]
V.8.A	0.571999	0.436901	80.248706	[6]
V.8.B	0.151718	0.529800	36.869925	[6]
V.11.A	0.202217	0.531104	53.620586	[4]
V.11.B	0.271263	0.513256	55.913821	[4]
V.12.A	0.158494	0.537300	56.943003	[6]
V.14.A	0.230004	0.532303	71.010663	[4]
V.15.A	0.187116	0.525972	70.741829	[6]
V.16.A	0.162366	0.530500	74.606560	[6]
V.17.A	0.210832	0.517410	80.322576	[4]
V.17.B	0.213273	0.516543	80.354813	[4]
V.17.C	0.213854	0.519867	81.216731	[4]
V.17.D	0.219373	0.517781	81.270278	[4]
V.17.E	0.227212	0.520048	82.670260	[4]
V.17.F	0.219977	0.523434	82.742320	[4]
V.17.G	0.226699	0.524624	83.785470	[4]
V.17.H	0.268638	0.522727	88.673234	[4]
V.17.I	0.260505	0.531169	89.940465	[4]
V.17.J	0.289904	0.522624	91.981778	[4]
V.20.A	0.158293	0.527887	92.177881	[6]

Table VIII: Initial conditions for periodic three-body orbits in Sequence VI (yarn).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
VI.2.A	0.464445	0.396060	14.894306	[3]
VI.4.A	0.462608	0.397137	29.706764	[6]
VI.6.A	0.559064	0.349192	55.502421	[3]
VI.6.B	0.558625	0.351335	55.714137	[6]
VI.6.C	0.558397	0.353353	55.960227	[6]
VI.12.A	0.559094	0.355302	112.839510	[6]
VI.14.A	0.394501	0.428999	94.011981	[6]

Table IX: Initial conditions for periodic three-body orbits in Sequence VIIa (moth).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
VIIa.6.A	0.442591	0.423514	35.833469	[6]
VIIa.7.A.α	0.494752	0.409250	47.201309	[6]
VIIa.7.A.β	0.473674	0.431289	46.885554	[6]
VIIa.10.A	0.434075	0.460697	64.121147	[6]
VIIa.11.A	0.458038	0.409375	66.351974	[6]
VIIa.13.A	0.505939	0.389401	86.418013	[6]
VIIa.13.B	0.492544	0.437327	95.073347	[6]
VIIa.13.C	0.490505	0.404421	85.385972	[6]

Table X: Initial conditions for periodic three-body orbits in Sequence VIIb (moth).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
VIIb.4.A	0.537956	0.341458	26.918667	[6]
VIIb.7.A	0.409495	0.362823	33.867761	[6]
VIIb.7.B	0.523394	0.342113	44.930815	[6]
VIIb.9.A	0.476366	0.378935	53.472480	[6]
VIIb.10.A	0.396049	0.352941	46.133172	[6]
VIIb.13.A	0.413880	0.347796	61.844743	[6]
VIIb.15.A	0.344750	0.393045	67.387774	[6]
VIIb.15.B	0.294621	0.415210	64.917046	[6]
VIIb.16.A	0.408991	0.345713	75.021111	[6]
VIIb.17.A	0.416066	0.297150	74.788574	[6]
VIIb.18.A	0.396743	0.370881	85.934381	[6]
VIIb.20.A	0.415757	0.298190	88.046020	[6]

Table XI: Initial conditions for periodic three-body orbits in Sequence VIII (other).

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	Ref.
VIII.4.A	0.201678	0.409896	21.020518	[6]
VIII.8.A	0.301500	0.441007	39.611755	[6]
VIII.10.A	0.268073	0.443797	48.894527	[6]
VIII.15.A	0.324059	0.366571	61.698083	[6]
VIII.15.B	0.288061	0.437072	74.392687	[6]

[1] http://three-body.ipb.ac.rs/sequences.php
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